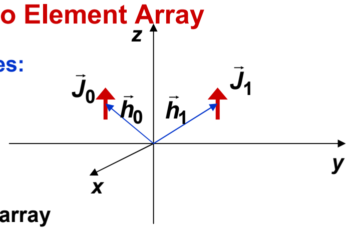
Lecture 33

Antenna Arrays and Phase Arrays

In this lecture you will learn:

- Antenna arrays
- Gain and radiation pattern for antenna arrays
- Antenna beam steering with phase array antennas

Two Hertzian Dipoles – A Two Element Array



Consider first an array of just two Hertzian dipoles:

$$\vec{J}_0(\vec{r}) = \hat{z}\, I_0\, d\, \delta^3(\vec{r} - \vec{h}_0)$$

$$\vec{J}_1(\vec{r}) = \hat{z}\, I_1\, d\, \delta^3(\vec{r} - \vec{h}_1)$$

Each antenna in the array is an "element" of the array

One can write the E-field in the far-field as a superposition of the E-fields produced by all the elements in the array:

$$\vec{E}_{ff}(\vec{r}) = \hat{\theta}\,\frac{j\,\eta_o\, kd}{4\pi\, r}\sin(\theta)\, e^{-j\,k\,r}\left[I_0\;\, e^{j\,k\,\hat{r}.\vec{h}_0} + I_1\;\, e^{j\,k\,\hat{r}.\vec{h}_1}\right]$$

$$= \underbrace{\hat{\theta}\,\frac{j\,\eta_o\, k\, I_0 d}{4\pi\, r}\sin(\theta)\, e^{-j\,k\,r}}_{\text{Element Factor}}\underbrace{\left[\frac{I_0}{I_0}\; e^{j\,k\,\hat{r}.\vec{h}_0} + \frac{I_1}{I_0}\; e^{j\,k\,\hat{r}.\vec{h}_1}\right]}_{\text{Array Factor}}$$

$$= \mathrm{E}(r,\theta,\phi)\;\; F(\theta,\phi)$$

- The "element factor" is just the E-field produced by the first element if it were sitting at the origin
- The "array factor" captures all the interference effects

An N-Element Antenna Array - I

Consider an N-element antenna array where the elements are:

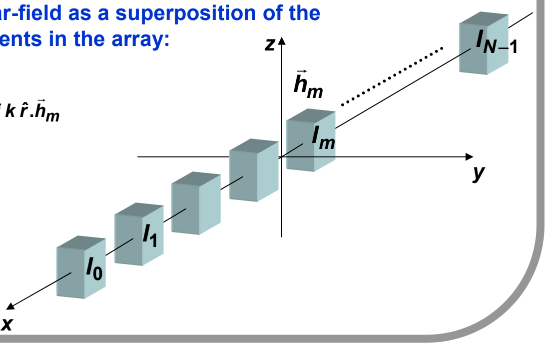
- **all identical (all loops, or all Hertzian dipoles, or all Half-wave dipoles, etc)**
- **all oriented in the same way**
- **but with possibly different current phasors**

Let the current phasor of the *m*-th antenna be I_m

Let the position vector of the *m*-th antenna be $\vec{h}_m$

One can write the E-field in the far-field as a superposition of the E-fields produced by all the elements in the array:

$$\vec{E}_{ff}(\vec{r}) = \vec{\mathrm{E}}(r,\theta,\phi)\sum_{m=0}^{N-1}\frac{I_m}{I_0}\,e^{j\,k\,\hat{r}.\vec{h}_m}$$
$$= \vec{\mathrm{E}}(r,\theta,\phi)\;F(\theta,\phi)$$

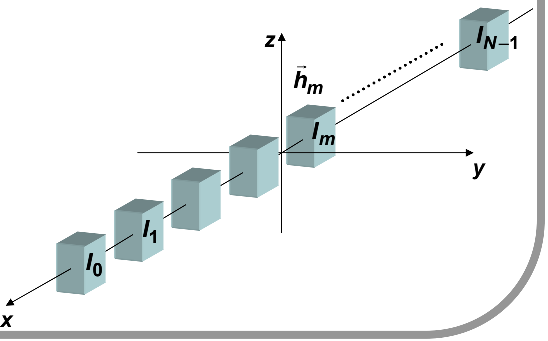


An N-Element Antenna Array - II

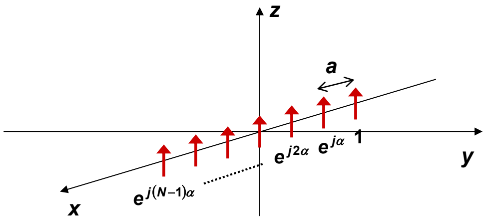
$$\vec{E}_{ff}(\vec{r}) = \vec{\mathrm{E}}(\vec{r})\sum_{m=0}^{N-1}\frac{I_m}{I_0}\,e^{j\,k\,\hat{r}.\vec{h}_m}$$
$$= \vec{\mathrm{E}}(r,\theta,\phi)\;F(\theta,\phi)$$

Gain: $$G(\theta,\phi) = \frac{\langle\vec{S}(\vec{r},t)\rangle.\hat{r}}{P/4\pi r^2} \propto \left[\text{angular part of } \left|\vec{\mathrm{E}}(r,\theta,\phi)\right|^2\right]\left|F(\theta,\phi)\right|^2$$

Pattern: $$p(\theta,\phi) = \frac{G(\theta,\phi)}{G(\theta,\phi)|_{\max}}$$



An N-Element Hertzian Dipole Phase Array



N-element Hertzian dipole array

- Current magnitude is the same for all the dipoles
- Current phase difference between successive dipoles is α

$$\frac{I_{m+1}}{I_m} = e^{j\alpha}$$

Element Factor:

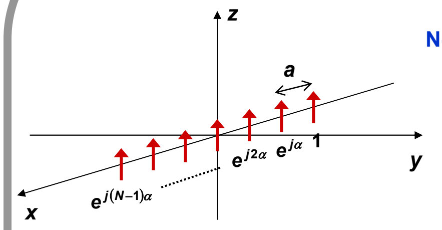
$$\vec{\mathrm{E}}(r,\theta,\phi) = \hat{\theta}\,\frac{j\eta_o k I_0 d}{4\pi r}\sin(\theta)\,e^{-jkr}$$

Array Factor:

$$F(\theta,\phi) = \sum_{m=0}^{N-1} \frac{I_m}{I_0}\,e^{jk\hat{r}.\vec{h}_m} = e^{jk\hat{r}.\vec{h}_0}\sum_{m=0}^{N-1} e^{j\alpha m}\;e^{jkam\sin(\theta)\cos(\phi)}$$

$$\Rightarrow\quad |F(\theta,\phi)|^2 = \left|\sum_{m=0}^{N-1}\left(e^{jka\sin(\theta)\cos(\phi)+\alpha}\right)^m\right|^2 = \frac{\sin^2\left[\frac{N}{2}(\alpha+ka\sin(\theta)\cos(\phi))\right]}{\sin^2\left[\frac{1}{2}(\alpha+ka\sin(\theta)\cos(\phi))\right]}$$

An N-Element Hertzian Dipole Phase Array



N-element Hertzian dipole phase array antenna

$$P = \frac{\eta_o}{12\pi}|k\,I_0\,d|^2 N$$

$$G(\theta,\phi) = \frac{\langle\vec{S}(\vec{r},t)\rangle.\hat{r}}{P/4\pi r^2} = \frac{3}{2N}\sin^2(\theta)\;|F(\theta,\phi)|^2$$

Coming from the element factor

array factor

Pattern:

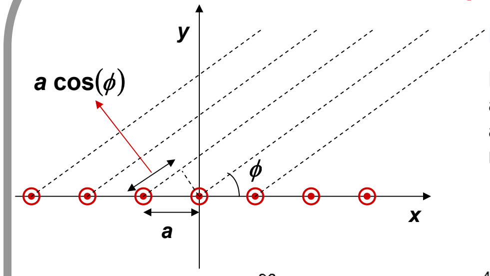
$$p(\theta,\phi) = \frac{1}{N^2}\sin^2(\theta)\;|F(\theta,\phi)|^2$$

$$p(\theta,\phi) = \frac{1}{N^2}\sin^2(\theta)\frac{\sin^2\left[\frac{N}{2}(\alpha+ka\sin(\theta)\cos(\phi))\right]}{\sin^2\left[\frac{1}{2}(\alpha+ka\sin(\theta)\cos(\phi))\right]}$$

Coming from the element factor

array factor

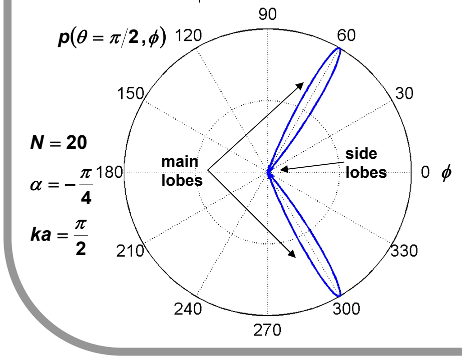
An N-Element Hertzian Dipole Phase Array: Maxima

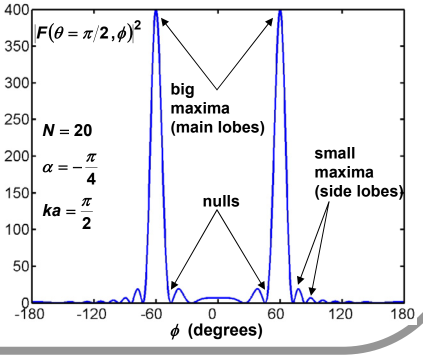


Lets look at radiation in the *x-y* plane:

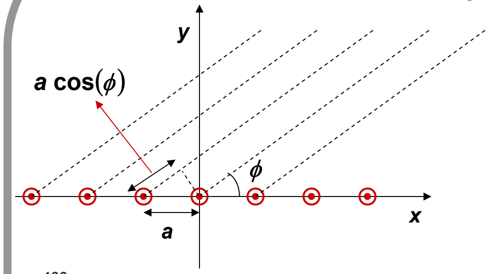
Radiation going in the ϕ direction from adjacent elements will add in-phase, and one will have a big maximum in the radiation pattern, provided:

$$k\,a\cos(\phi)+\alpha=\pm 2\pi\,n \quad \{\, n=0,1,2,3,\ldots\ldots$$





An N-Element Hertzian Dipole Phase Array: Maxima



Condition for a big maximum in the *x-y* plane:

$$k\,a\cos(\phi)+\alpha=\pm 2\pi\,n \quad \{\, n=0,1,2,3,\ldots\ldots$$

At a big maximum the value of the array factor is:

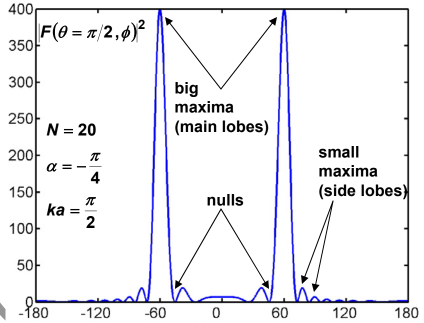
$$|F(\theta=\pi/2,\phi)|^2_{\max}=\frac{\sin^2\left[\frac{N}{2}(\alpha+ka\cos(\phi))\right]}{\sin^2\left[\frac{1}{2}(\alpha+ka\cos(\phi))\right]}$$

$$=N^2$$

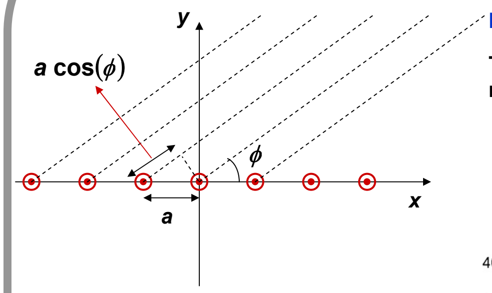
And the value of the antenna gain is:

$$G(\theta,\phi)=\frac{3}{2\,N}\sin^2(\theta)\;|F(\theta,\phi)|^2$$

$$\Rightarrow G\left(\theta=\frac{\pi}{2},\phi\right)_{\max}=\frac{3}{2}N$$



An N-Element Hertzian Dipole Phase Array: Nulls

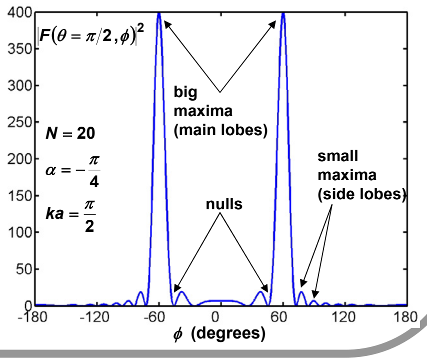


Radiation in the *x-y* plane:

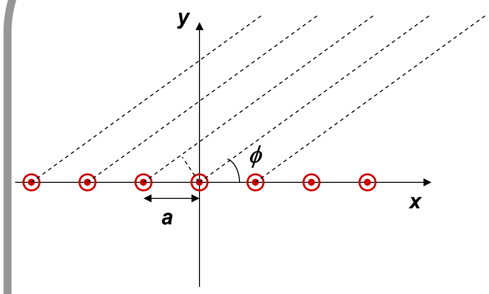
The array factor will give a null in the radiation pattern provided:

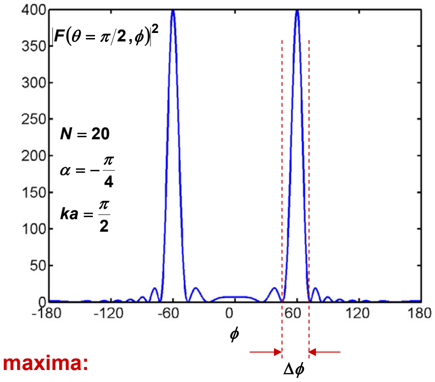
$$k\,a\cos(\phi)+\alpha=\pm\frac{2\pi\,n}{N}\qquad\{\;n\neq 0,N,2N,3N,\ldots\ldots$$

$$\left|F(\theta=\pi/2,\phi)\right|^2=\frac{\sin^2\left[\frac{N}{2}\left(\alpha+ka\cos(\phi)\right)\right]}{\sin^2\left[\frac{1}{2}\left(\alpha+ka\cos(\phi)\right)\right]}$$



An N-Element Hertzian Dipole Phase Array: Width of Big Maxima





Angular width of a lobe associated with a big maxima:

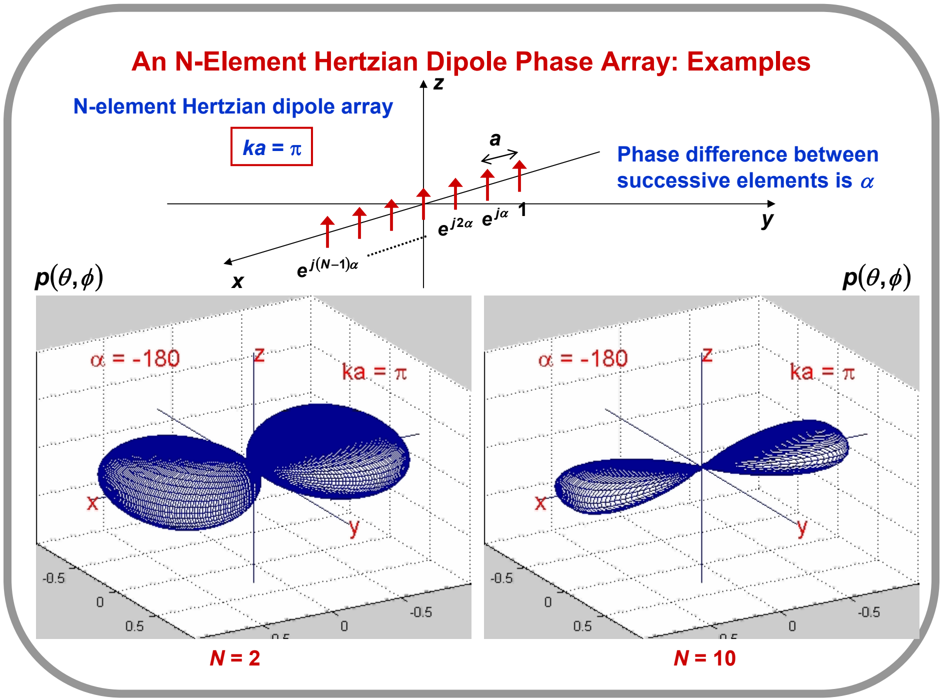
For a big maxima: $$k\,a\cos(\phi)+\alpha=2\pi\,n=2\pi\frac{nN}{N}\qquad\{\;n=0,\pm1,\pm2,\pm3,\ldots\ldots$$

At the nulls nearest to the above maxima we must have:

$$k\,a\cos\left(\phi+\frac{\Delta\phi}{2}\right)+\alpha=2\pi\frac{nN\pm1}{N}$$

Which can be solved for $\Delta\phi$ – and for N large ($N >> 1$) one gets approximately:

$$\Delta\phi\approx\left|\frac{4\pi}{N}\;\frac{1}{ka\sin(\phi)}\right|$$



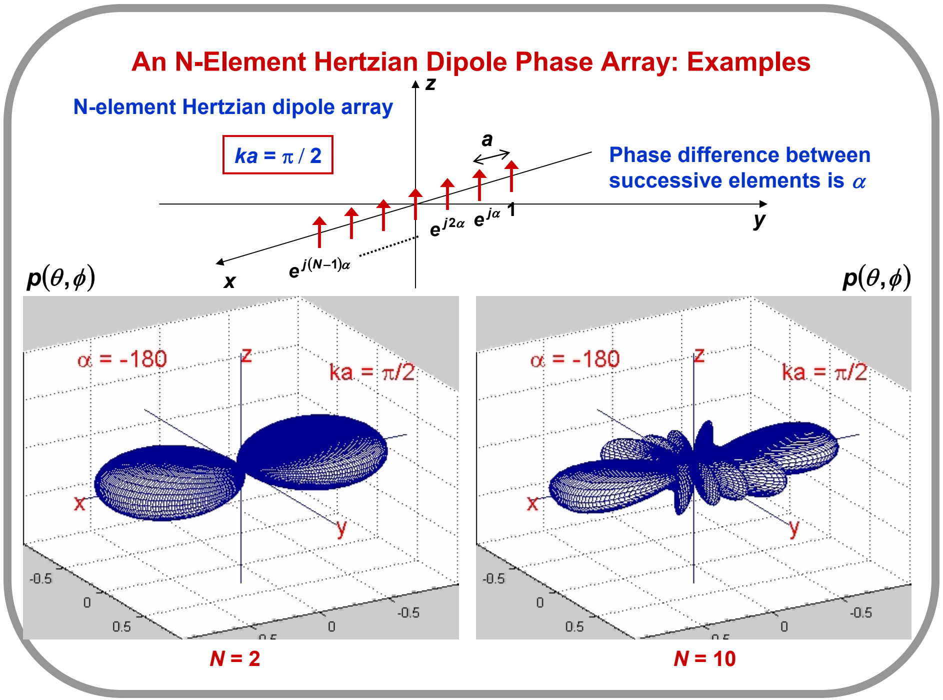
An N-Element Hertzian Dipole Phase Array: Examples

N-element Hertzian dipole array

$\boxed{ka = \pi}$

Phase difference between successive elements is α

***N* = 2** **N = 10**



An N-Element Hertzian Dipole Phase Array: Examples

N-element Hertzian dipole array

$\boxed{ka = \pi / 2}$

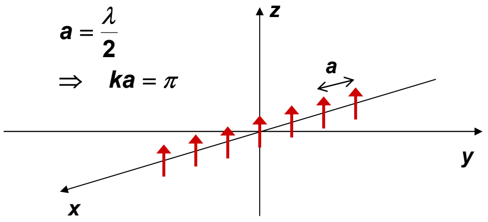
Phase difference between successive elements is α

***N* = 2** ***N* = 10**

An (N+1)-Element Hertzian Dipole Binomial Array

$$a=\frac{\lambda}{2}$$

$$\Rightarrow\quad ka=\pi$$



(N+1)-element Hertzian dipole array

- Current phase is the same for all the dipoles
- Currents magnitudes in the dipoles follow a binomial distribution

$$\frac{I_m}{I_0}=\frac{N!}{m!\,(N-m)!}$$

Element Factor:

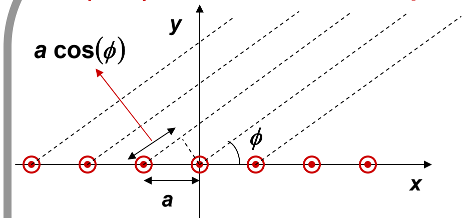
$$\vec{E}(r,\theta,\phi)=\hat{\theta}\,\frac{j\,\eta_o\,k\,I_0 d}{4\pi\,r}\sin(\theta)\,e^{-j\,k\,r}$$

Array Factor:

$$F(\theta,\phi)=\sum_{m=0}^{N}\frac{I_m}{I_0}\,e^{j\,k\,\hat{r}.\vec{r}_m}=e^{j\,k\,\hat{r}.\vec{r}_0}\sum_{m=0}^{N}\frac{N!}{m!\,(N-m)!}\,e^{j\,k\,a\,m\sin(\theta)\cos(\phi)}$$

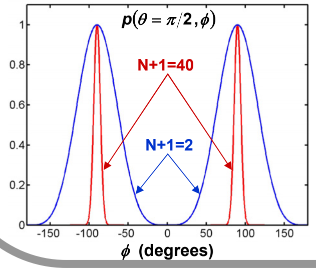
$$\Rightarrow\quad\left|F(\theta,\phi)\right|^2=\left|1+e^{j\,k\,a\sin(\theta)\cos(\phi)}\right|^{2N}=2^{2N}\cos^{2N}\left[\frac{k}{2}a\sin(\theta)\cos(\phi)\right]$$

An (N+1)-Element Hertzian Dipole Binomial Array: Maxima and Nulls



$$\left|F(\theta=\pi/2,\phi)\right|^2=2^{2N}\cos^{2N}\left[\frac{\pi}{2}\cos(\phi)\right]$$

$$p(\theta=\pi/2,\phi)=\cos^{2N}\left[\frac{\pi}{2}\cos(\phi)\right]$$



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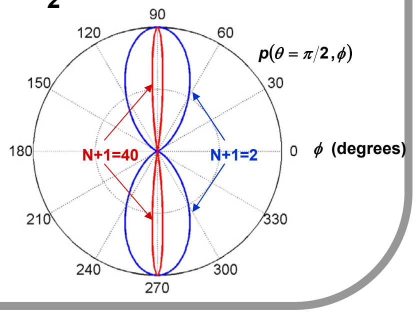
$$\{\,n=0,1,2,3,\ldots\ldots$$

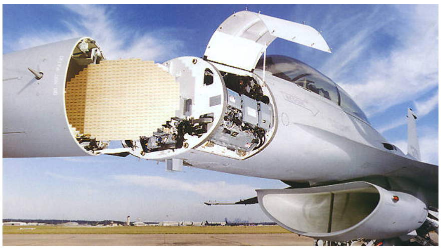
$$\Rightarrow\quad\frac{\pi}{2}\cos(\phi)=\pm\pi\,n$$



Note:

- **No side lobes**
- **Two nulls at φ = 0,π**

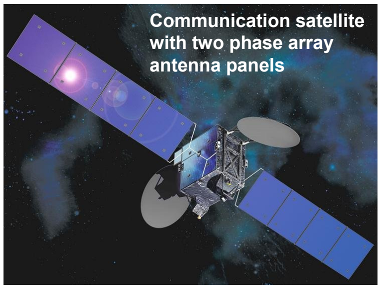
Applications of Antenna Arrays - I



Phased Array TRack to Intercept Of Target

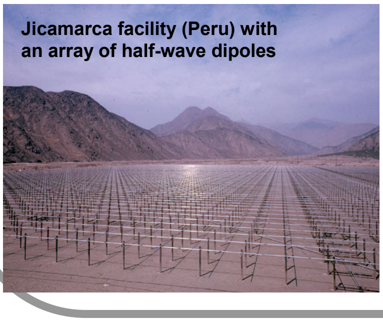
Alaska

Applications of Antenna Arrays - II

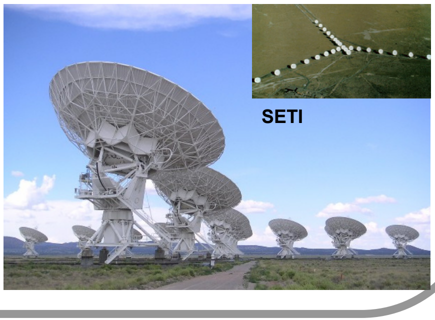


Communication satellite with two phase array antenna panels

MRI system with phase array RF receivers



Jicamarca facility (Peru) with an array of half-wave dipoles



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